Local and Nonlocal Models in Materials Science

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Nonlocal theories have been introduced in the mechanics of solids, due to the appearance of cracks and other discontinuities which hinder the use of classical differential operators. The use of nonlocal operators has been proven valuable in many other areas, including image processing [4, 5], sandpile formation [1], swarm [10] and other population density models [2].

The recent theory of peridynamics, introduced by Silling in 2000 in [11], prescribes that spatial derivatives may be avoided in modeling the interactions between particles. In this nonlocal version of continuum theory, internal forces acting on material particles are assumed to form a network of pairwise forces, known as bonds. Each point \( x \) interacts with the neighboring points inside a region of radius \( \delta \) (called horizon) centered at \( x \). With this setup fracture is seen as breaking of the aforementioned pairwise bonds, and hence it can be modeled by a vanishing pairwise force function. Thus the strength of the peridynamic formulation is that the same equations can be applied to all points in the domain, eliminating the previous need for special techniques from fracture mechanics.

The peridynamic theory may be thought of as a continuum version of molecular dynamics and in this theory the acceleration of any particle at \( x \) in the reference configuration at time \( t \) is found from the equation of motion

\[
(0.1) \quad \rho(x)u_{tt}(x,t) = \int_{H_x} f(u(x',t) - u(x,t), x' - x)dx' + b(x,t),
\]

where \( H_x \) is a neighborhood of \( x \), \( u \) is the displacement vector field, \( b \) is a prescribed body force density field, \( \rho \) is mass density in the reference configuration, and \( f \) is a pairwise force function whose value is the force vector that the particle \( x' \) exerts on the particle \( x \). It is convenient to assume that for a given material there is a positive number \( \delta \), called the horizon, such that

\[
|\xi| > \delta \Rightarrow f(\eta, \xi) = 0, \text{ for every } \eta.
\]

Hence \( H_x \) (the domain of interaction between \( x \) and its neighbors) is taken as the ball centered at \( x \) of radius \( \delta \).

The theory has been used recently in wave propagation in media with fractures, and in predicting fractures in elastic bodies [11, 12]. Numerical computations [6] show a very good agreement of the theory with observations, drastically improving previous results, thus making it a very promising area for applications and research.
However, the mathematical theory is still in its early stages of development. The participants in this project will work to:

(1) Find appropriate spaces and topologies for the study of the integro-differential equation (0.1), for different choices of the forcing \( f \), the most common one being the linear formulation given by

\[
f(u(x', t) - u(x, t), x' - x) = \gamma(x' - x)(u(x', t) - u(x, t)),
\]

for kernels \( \gamma \) that are integrable (weakly singular).

(2) Show existence of solutions for (0.1) and study their qualitative properties; preliminary results were obtained in [7, 3] covering the linear and nonlinear realms;

(3) Obtain results that show the connection between peridynamics and continuum mechanics (preliminary results were obtained in [9, 8] but the mathematical framework is in its incipient phases).

References


